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Jiangmei Wu is an interdisciplinary scholar who creates art and design projects that involve mathematics, science, and engineering. Recently, she has been exploring the relationship between geometry, surface texture, computational algorithms, and making techniques in the art and science of paper folding. Her origami-inspired, large-scale installations have been exhibited in museums across the United States as well as in museums and galleries in China, Italy, Japan, the Netherlands, Denmark, and Thailand. She has won several art and design awards, including the American Mathematical Society's award for the best artwork in 2017 and an honorable mention in 2023 in the category of textile, sculpture, or other medium. She also holds three U.S. patents for her innovative design techniques. Her academic papers have been published in several leading research journals in the fields of architecture, interior design, and mathematical art. In addition, Wu has collaborated on NSF funded research in STEAM education for engineering and design students.

The Artistry and Algorithms in Fabric Origami Tessellations

Abstract: This article describes artistic and algorithmic strategies in making fabric origami to create fabric art with tactile sensitivities and expressive shadow effects.

Keywords: fabric manipulation, origami, tessellation, crease pattern, algorithm, textile art, tessellation grafting

1. Introduction

Surface texture in fashion design and textile art is often achieved through fabric manipulation techniques such as gathering, shirring, pleating, smocking, or tucking. This article describes how, as an artist/designer, I used mathematical algorithms to create surface texture patterns for fabric origami art using sewing, gathering, and pleating. The term "fabric origami" refers only to certain fabric manipulation techniques that alter the texture of the original fabric to create a new fabric that is endowed with novel visual and tactile characteristics. A variety of fabric materials can be used for fabric origami, including natural ones such as cotton and silk, and material made from polyethylene by DuPont. Figure 1 shows a large fabric origami of this material; the black design with circles on one side represents the artwork as well as the sewing patterns while the textured fabric origami is on the other side.

Through fabric origami it is my hope that the often-underappreciated precision, patience, and techniques that go into its production can be seen as a metaphor for the unacknowledged skills and labor often associated with women's labor. The transformative process of turning a flat piece of fabric into a complex textured form can symbolize women's transformation and empowerment.

Figure 1. *Step into the Cloud, 300"x30", Tyvek 14, 2023. Installation views as part of an international exhibition at Museum for Papirkunst, Blokhus Denmark. The fabric origami sewing pattern is incorporated into the black dotted design printed on one side of the Tyvek sheet. The other side shows the textured fabric origami.*

2. Fabric Manipulation and Fabric Origami Tessellation

Fabric manipulation can be defined as the modification of a basic fabric to create a new innovative fabric surface that is more visually and tactilely different than the original (Burns, 2022). Of all the fabric manipulation techniques, systematic folding, such as gathering and pleating, is similar to origami manipulations in that it reduces the original size of the fabric by certain proportions. Pleating and gathering involves measured folds on a piece of fabric. This can be done with simple pleats or with complex tessellations (Wolff, 1996). Folds can be pressed with an iron or treated with heat to create sharp edges or be left unpressed to allow the fabric to project and curve naturally. Today, fabric gathering and pleating is often done by machine and treated with heat, making pleats more accessible to a large population. Many contemporary fashion designers, including Germaine Émelie Krebs and Issey Miyake have used pleated fabric in their collections (Burns, 2022). Pleating fabric is sometime called *orinuno* -- in Japanese, *Ori* means fold and *Uno* means fabric. Artist Reiko Sudo's origami corrugation pleats and other more complicated pleated tessellations can be seen in scarves collected by major museums around the world (Museum, 2013).

A tessellation (or tiling) is traditionally described as the filling of a flat surface by one or more shapes (called tiles) so that there are no gaps or overlaps. Origami tessellations begin with a sheet (usually of paper) that is folded according to a crease pattern to produce a pattern of shapes that resembles a tiling by tiles. An origami tessellation is often a design in which both the crease pattern and the folded structures use continuous and repeated elements. Origami tessellations were first developed by Yoshihide Momotani (Momotani, 1984) and Shuzo Fujimoto (Fujimoto & Nishiwaki, 1982), and their mathematical properties were first studied by Toshikazu Kawasaki and Masaaki Yoshida in the context of crystallographic symmetry (Kawasaki & Yoshida, 1988). In the 1960s and 70s, Ronald Resch and David Huffman each created many interesting origami tessellations. Recently, many origami artists, including Chris Palmer (Palmer, 1997), Eric Gjerde (Gjerde, 2009), Yuko Nishimura, Goran Konjevod, Ray Schamp, Alessandro Beber (Lang, 2018), Helena Verrill (Verrill, 1998), and others have pushed the art form to new heights, establishing a new abstract origami genre that is distinctive from the traditional figurative origami. This new interest in origami tessellations has led artists, mathematicians, and computer scientists within the origami community to search for new origami tessellation designs by studying mathematical tiling and developing computer algorithms. For example, Alex Bateman's computer program *Tess* allows users to create origami tessellations based on mathematical tilings such as Archimedean tilings (Bateman, 2007). More recently, Robert Lang has discussed mathematical concepts underpinning origami tessellations and ways to design origami tessellations (Lang, 2018).

Origami tessellations are typically folded using paper imprinted with a complex crease pattern made of alternating mountain and valley folds. To fold an origami tessellation, one needs to gently fold and unfold creases in order to collapse and flatten individual creases—a process that may take hours to complete. There is always a risk of the paper failing or being torn due to this demanding process. Certain tessellations, such as the ones involving twists, are very difficult to fold in paper. To overcome the problem, some artists have started to use fabric. Origami artists Jefferey Rustzky and Chris Palmer have used this technique to create a wide array of tessellation designs, called "Shadowfolds," to be used as decorative textiles (Rutzky & Palmer, 2011).

3. Introduction to Fabric Origami

According to Jefferey and Chris Palmer, the techniques for making fabric origami are very simple (Rutzky & Palmer, 2011). First, a sewing pattern is transferred to a piece of fabric, then the points are sewn together to allow the fabric to gather. Finally, the fabric is flattened to form origami tessellation designs such as pleats and/or twists.

Figure 1 shows a simple example of the process of making a fabric origami tessellation. A *sewing pattern* in fabric origami is a diagram that shows points and lines used to guide the sewing process. The points show where the fabric needs to be stitched and the lines between the points indicate how these points need to be tied together. The sewing pattern can be transferred onto the fabric either by hand or inkjet printing.

In Figure 1(a), the sewing pattern is transferred to a piece of muslin fabric using ink and pencil to create dots that are then connected with triangles or lines. To sew the fabric, a small fine needle is threaded with a single strand of thread that matches the fabric's color. One of the dots that are connected by a triangle is picked up by the needle and the thread is fastened by a dead knot tied at that location. The next two dots in the triangle are pierced by picking up a few threads in the fabric (Figure 2(b)) and then the thread ends are pulled together to gather all the dots to one point. The thread is then fastened (knotted) at that point so the thread in the needle doesn't move (Figure 2(c)). Without cutting the thread, the needle is moved to one of the dots in an adjacent triangle or line while leaving enough thread length between the tied knot and the next triangle or line. The same process is repeated until all triangles or lines are tied together on the same side of the fabric. If there is any fabric protruding on this side, it is simply tucked to the other side (Figure 2(d)). The threads between the tied points are then cut. This side of the fabric will lay flat on the plane due to the design of the sewing pattern (generating sewing patterns is discussed in section 4). Turning the fabric over, the gathered fabric on the other side is nearly ready to be pleated into twists or flaps (2(e)). The gathered fabric is lightly pinched with fingers to form a clean fold and then gently pressed down to form a pleat. This process is continued until all the gathered fabric is pleated flat. There are multiple ways to form the pleats (how to form the pleats is discussed in section 5).

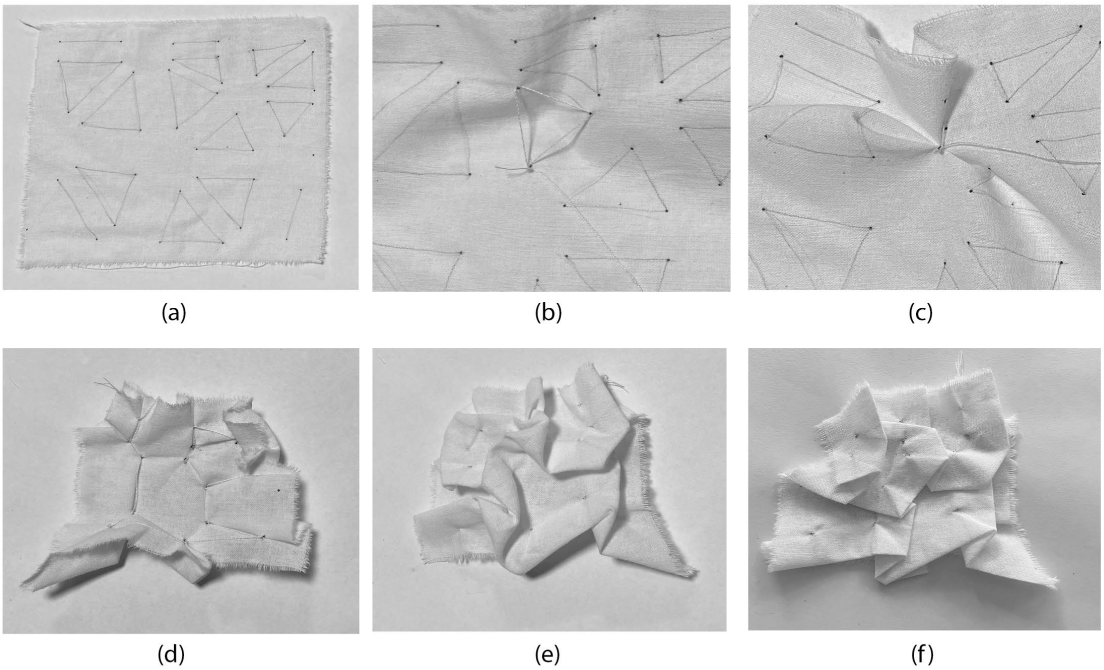


Figure 2. *A simple example showing the fabric origami process.*

While Figure 2 shows an example of fabric origami in which all the ties are made on one side of the fabric to allow the fabric to be pleated on the other side, one can also sew and tie the fabric on either side to allow the fabric to be pleated on both sides. Figure 3 shows two such examples that are created based on the same sewing pattern. Figure 3a shows an example in which all the points of each of the rhombi are sewn and tied on one side of the fabric while the fabric is gathered and pleated on the other side. Figure 3b shows an example in which the top and bottom rows of rhombi are sewn and tied on one side of the fabric, and the middle row of rhombi are sewn and tied on the other side of the fabric. By allowing the fabric to gather and be pleated on both sides, interesting origami tessellations can be generated.

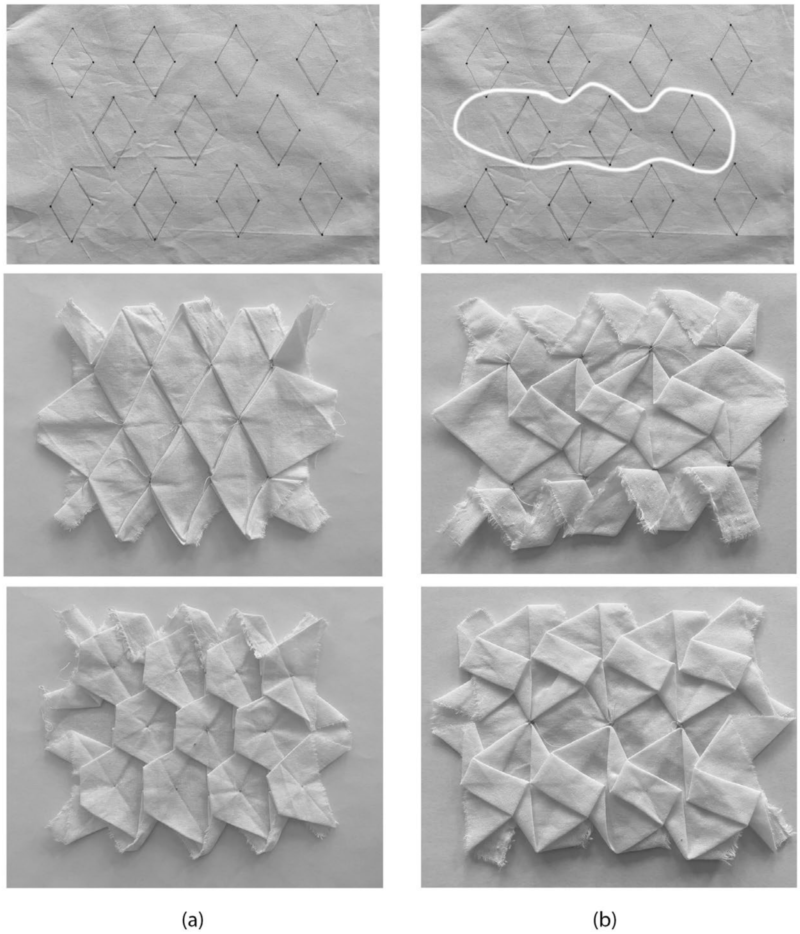


Figure 3. *Two different examples of pleating using the same sewing pattern.*

4. Tessellation Grafting Technique for Generating Sewing Patterns

The key to interesting fabric origami designs is to create the sewing patterns. *Tessellation grafting* is a method I devised to generate sewing patterns for fabric origami tessellations. In what follows, a tessellation will refer to an edge-to-edge tiling by polygons (regular or irregular). Edge-to-edge means that each edge of a polygon tile meets exactly one edge of another (and so each vertex of the tiling must also be a vertex of each of the polygon tiles that meet there). In addition, all tessellations discussed here will be finite, consisting of a finite number of polygon tiles, since all will be considered to produce patterns for origami fabric art pieces.

To "graft" such a tessellation, one starts by figuratively cutting apart adjacent polygons that share an edge and then expanding the space between them to allow rectangles and/or parallelograms to join the previously adjacent polygons. In addition, new polygons are inserted around the vertices of the original tessellation. If a vertex in the original tessellation has a valence of three, the polygon connecting the now separated vertices must be a triangle. If a vertex in the original tessellation has a valence of four, the polygon connecting the new vertices must be a quadrilateral. In general, for vertices that are *n*-valent, the inserting polygons must be *n*-gons.

Figure 4(a) shows a patch of the rhombille tessellation, made up of congruent 60º-120º rhombi, and the expansion of two vertex figures of the tessellation. To produce the expansions, the rhombi are scaled (dilated) at their geometric center by an arbitrary scale factor of 0.5 (Figure 4(b). (We will discuss the relationship between the scale factor and the sizes of fabric origami later in an example in section 7.) After the scaled rhombus tiles are spaced apart (Figure 4(c)), lines are drawn between their previously adjacent vertices to create rectangles that are shaded in gray. These lines also create new polygons in the space around the vertices of the original tessellation—a regular hexagon replaces each degree-6 vertex, and an equilateral triangle replaces each degree-3 vertex (Figure 4(d)).

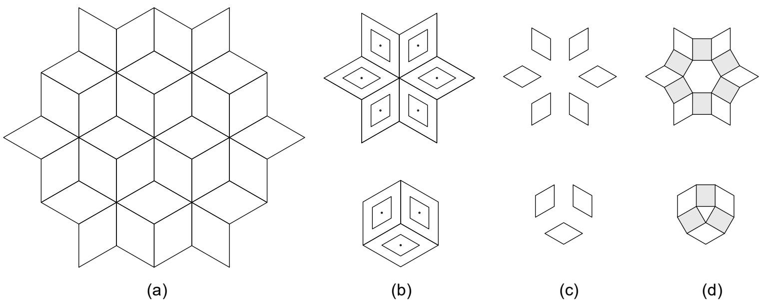


Figure 4. *An example showing tessellation grafting process.*

This process produces the grafted tessellation pattern in Figure 5 for a fabric origami. In Figure 5(a), and in subsequent figures in this article, polygons in the original tessellation are drawn in dashed lines, and solid lines are new lines drawn in the grafted tessellation. The polygons shaded in gray are the rectangles (or parallelograms) inserted during the tessellation grafting process. The sewing diagram in Figure 5(b) is the result of eliminating the original rhombi and the inserted rectangles. To make the fabric origami from the sewing pattern, the corners of each of the triangles are sewn together and the corners of each of the hexagons are sewn together on the front side of the fabric, collapsing the triangles and hexagons back to points and the rectangles back to lines. On the same front side, the fabric is carefully pleated and flattened to create a new tessellation (Figure 5(c)). On the back side of the sewn fabric the seams are now reflecting the original rhombille tiling (Figure 5(d)).

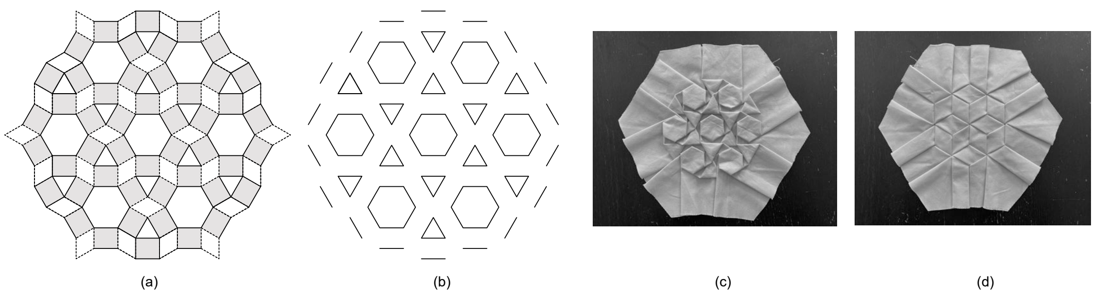


Figure 5. *A simple fabric origami example showing the grafted tessellation, its related sewing pattern, and the sewn result.*

Figure 6 shows another example of a tessellation patch that consists of 45º-135 º rhombi and squares. The polygons in the original patch (Figure 6(a)) are scaled at a ratio of about 0.55 to add space between the rhombi and squares. Notice that the spaces between rhombi are filled with congruent gray rectangles and the spaces between rhombi and squares are filled with gray parallelograms (Figure 6(b)). Figure 6(c) shows the sewing pattern made by eliminating the squares and rhombi. Figure 6(d) shows a sewn result in which sewing is done only on one side of the fabric, and Figure 6(e) shows another sewn result in which sewing is done on both sides of the fabric.

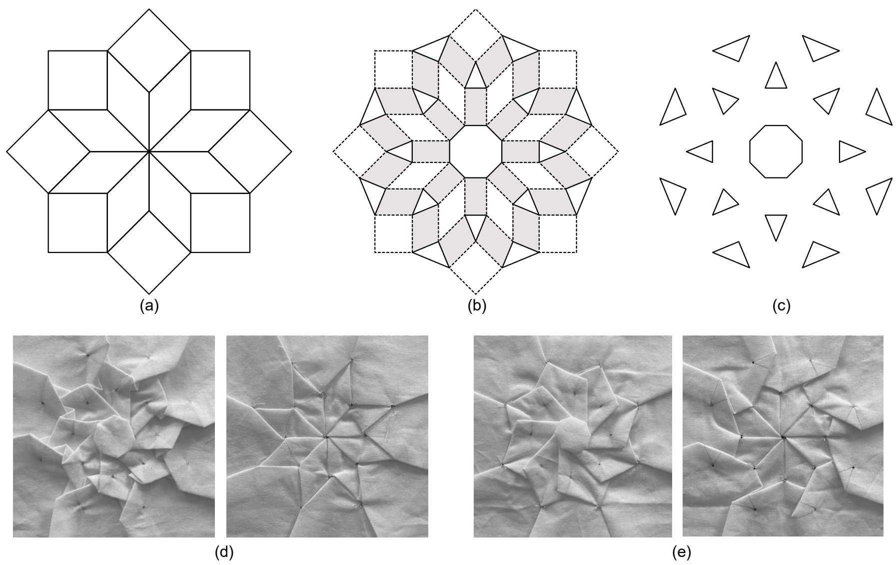


Figure 6. *Another simple fabric origami example showing the original tessellation, the grafted tessellation, its related sewing pattern, and the sewn results.*

So, one might ask, what kinds of grafted tessellations result in sewing patterns that can be sewn and pleated flat for fabric origami? What is unique about the examples shown in Figure 5 and Figure 6? How can the fabric be pleated? To understand these questions, the crease patterns generated from the sewing patterns must be studied.

5. Pleating techniques and Algorithms for Finding Flat-foldable Pleating Patterns

To understand how to pleat fabric flatly, the topic of flat-foldability of crease patterns of fabric origami must be studied first. The local flat-foldability at a single vertex has been very well studied (Lang, 2018). Here we reiterate several important theorems regarding this subject.

• *The Kawasaki-Justin Theorem*: A crease pattern is locally flat-foldable if and only if the number of crease lines connecting to a single inner vertex (a point where crease lines meet) is even and the sums of alternating angles between the lines is 180° (Justin, 1986; Kawasaki, 1989).

• *The Maekawa-Justin Theorem*: For any flat-foldable vertex, the difference between the numbers of the mountain folds and the number of valley folds is two (Justin, 1994).

• *The Big-Little-Big Angle Theorem (BLBA)*: For any flat-foldable vertex, the crease on either side of any sector whose angle is smaller than those of its neighbours must have opposite crease assignments (Lang, 2018).

In what follows, I will present a few algorithms for generating crease patterns from sewing patterns in fabric origami. In all the crease patterns presented, the solid lines are used to represent mountain folds, dashed and dotted lines are used to represent valley folds, and the original sewing patterns are superimposed in lighter lines and noted with uppercase letters. Unless otherwise noted, all the crease patterns presented satisfy the three theorems noted above. I leave it to the reader to provide proofs in each case.

To sew a single line in fabric origami, its end points are sewn together on one side of the fabric and the fabric is pleated and flattened on the other side. Pleats are created by folding according to crease patterns. Given a sewing diagram of a single line AB, as in Figure 7(a), the algorithm for generating the crease pattern is the following: Draw a perpendicular bisector of line AB. At either point A or point B, depending on which side the fabric is pleated, draw a line that is parallel to the perpendicular bisector of line AB. Assign the mountain or valley folds to the lines according to the way the fabric is pleated (folded to the left or the right). To fix the pleat, an additional sewing line CD that is congruent and parallel to AB can be added so that AB and CD form a rectangle (Figure 7(b)). However, a different sewing line CD can form a parallelogram with AB in which triangles ACB and BCD are congruent isosceles triangles. The algorithm for this crease pattern is as follows (Figure 7(c)): let C be on the perpendicular bisector of AB and draw another line going through point B and perpendicular to line AB. Draw CD perpendicular to this new line, so that CD is congruent and parallel to AB. Draw lines between A and C, B and C, and B and D. Then use the Maekawa-Justin Theorem and Big-Little-Big Angle Theorem to assign mountain and valley folds for these lines. However, if line AB and line CD are sewn on the opposite sides of the fabric, as shown in Figure 7(d), the resulting pleat has the same appearance as in Figure 7(a) and Figure 7(b). It is worth noting that if the area formed by sewing lines AB and CD is not a parallelogram and has no similar characteristics as the ones in Figure 7, no known flat-foldable crease patterns can be found.

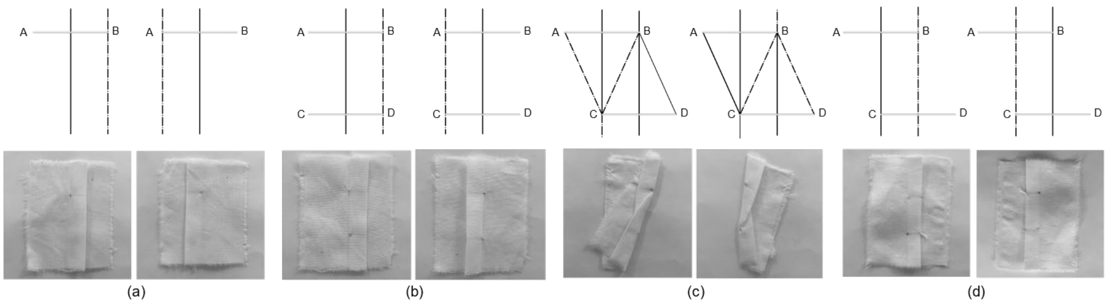


Figure 7. Crease patterns for sewing lines to fix pleats, and related sewn examples.

For sewing a single triangle in fabric origami, corners of the triangle are sewn to a point on one side of the fabric and then the fabric is pleated and flattened on the other side. There are several ways to pleat and flatten the fabric of a single triangle fabric origami, each of which corresponds to a crease pattern. Given a sewing diagram of a single triangle ABC with circumcenter O, the algorithm for generating the crease pattern shown in Figure 8 is the following: Draw perpendicular bisectors OD, OE, and OF of sides CB, BA, and AC, respectively. At point A, draw line AA″ so that AA″ is parallel to line OF. Let line AA″ intersect line OE at point A′. Repeat this for point B with line BB″ and point B′ and point C with line CC″ and point C′. Draw lines A′C′, C′B′, and B′A′ (Figure 8(a)). Finally, erase lines OA′, OB′, and OC′ (Figure 8(b)). Vertices A′, B′, and C′ satisfy the Kawasaki-Justin Theorem. Use the Maekawa-Justin Theorem and Big-Little-Big Angle Theorem to assign the mountain and valley folds at vertices A′, B′, and C′. The photo in Figure 8(b) shows the folded form. Figure 8(c) shows an alternative crease pattern generated from sewing triangle ABC and its resulting folded form.

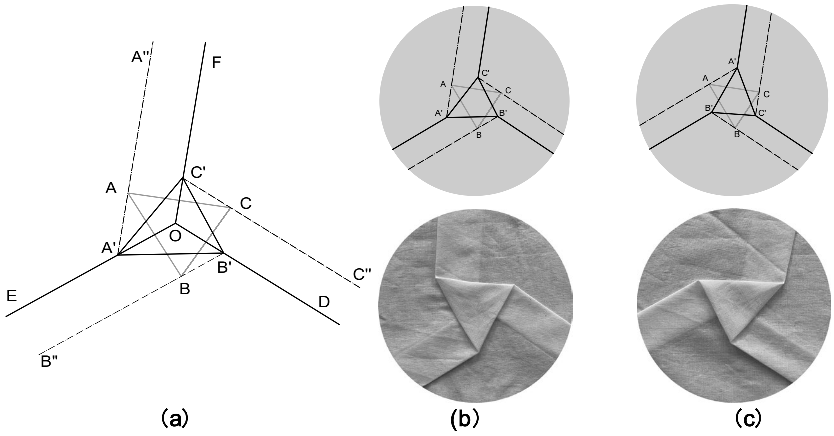


Figure 8. *Crease Patterns from sewing and then pleating a single triangle.*

Since using the algorithm described for Figure 8 will result in three pleats in which mountain creases and valley creases are parallel to each other, and there are two ways to fold each pair of pleats (the valley folds can be on either side of the mountain folds), the total possible crease patterns will be 2^3. In addition to the two crease patterns shown in Figure 8, Figure 9(a)–(f) shows the other six possible crease patterns and their respective folded forms. It should be noted these crease patterns are not new; Robert Lang called these patterns close-back twists and he generated them using a very different technique (Lang, 2018).

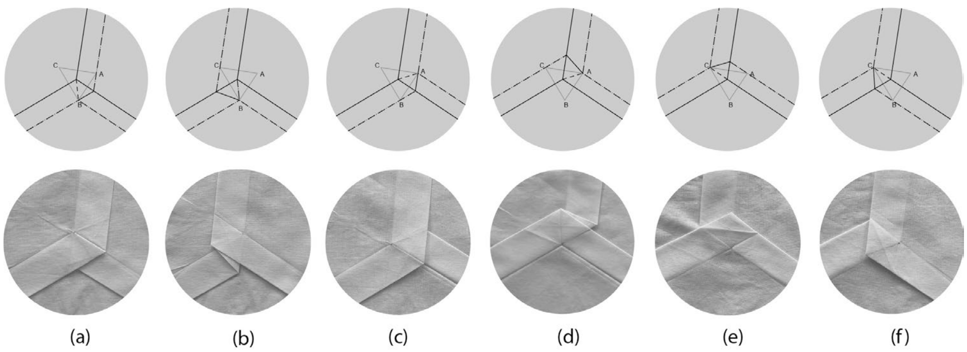


Figure 9. *Six alternative crease patterns from a single triangle and their respective folded forms.*

The crease patterns in Figures 8 and 9 share a common feature in that there are parallel pairs of mountain and valley creases. Due to this quality, these crease patterns can be tiled to generate flat-foldable tessellations. Since crease patterns can interact with each other in folded forms, determining the global flat-foldability of these tessellations is very difficult and beyond the topic of this paper. However, since there are eight possible flat-foldable crease patterns that can be assigned to each sewing triangle, it is possible for an artist to pleat and flatten a piece of fabric origami so that the pleats can be organized in a visually consistent way without intersecting with each other. Figure 10 shows an example of one of the many possible crease patterns generated from a sewing pattern of six triangles in fabric origami. Figure 10(a) shows the sewing pattern and Figure 10(b) the grafted tessellation with rectangles inserted between the triangles. Figure 10(c) shows one of the crease patterns that is a combination of sewing triangles and sewing lines (as in Figures 7 and 8), and Figure 10(d) its resulting folded form. The artist chose the pleating patterns based on both aesthetic and technical considerations.

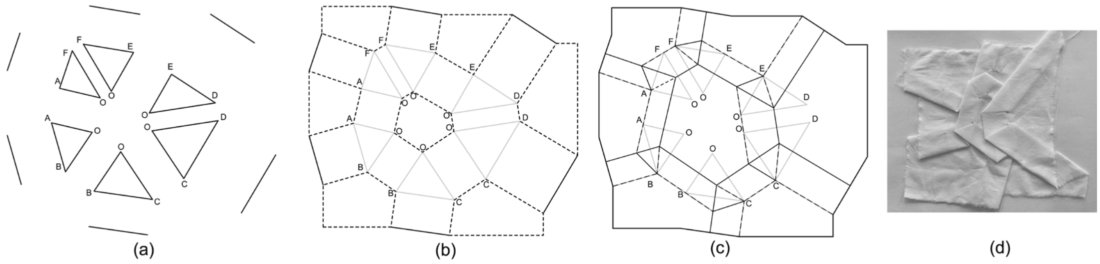


Figure 10. *An example of generating a crease pattern from six sewing triangles.*

Though the parallelism found in pairs of mountain and valley folds might be a good feature for creating tessellated crease patterns as shown in Figure 10, the parallelism is not a necessary condition for local flat-foldability for sewing one triangle. Given a sewing diagram of a single triangle ABC with an *arbitrary* interior point O, an alternative algorithm for generating a crease pattern from a sewing pattern is the following: Find circumcenters A′, B′, C′ for triangles AOC, AOB, and BOC respectively. Connect points A and A′ and then extend to create line AA″. Repeat this process with point B and B′, and C and C′ to create lines BB″ and CC″. Draw the perpendicular bisector EB′ of edge AB, DC′ of edge BC, and FA′ of edge CA (Figure 11(a)). Erase lines OA, OB, and OC. Vertices A′, B′, and C′ satisfy the Kawasaki-Justin Theorem. Use the Maekawa-Justin Theorem and Big-Little-Big Angle Theorem to assign the mountain and valley folds at vertices A′, B′, and C′. Figures 11(b) and 11(c) show the resulting crease pattern and the folded form. Since point O is arbitrary, there are infinite possibilities for flat-foldable crease patterns generated using this algorithm.

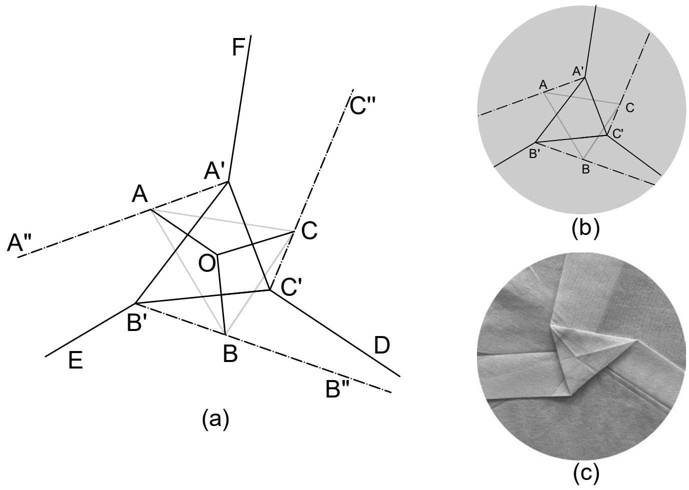


Figure 11. A *crease pattern from sewing a single triangle.*

While the crease patterns generated using the algorithm in Figure 11 might not be useful in creating triangular tessellations, the algorithm is useful for generating crease patterns from other sewing patterns such as the fabric origami example shown in Figure 3(b). Figure 12(a) shows its grafted tessellation with the 60º-120º rhombi ABCD. Notice that the grey shaded spaces are also 60º-120º rhombi, which are parallelograms made of two equilateral triangles. Figure 12(b) shows the crease pattern for the fabric origami in Figure 3(b), in which the alternative rows of rhombi are sewn on alternative sides of the fabric.

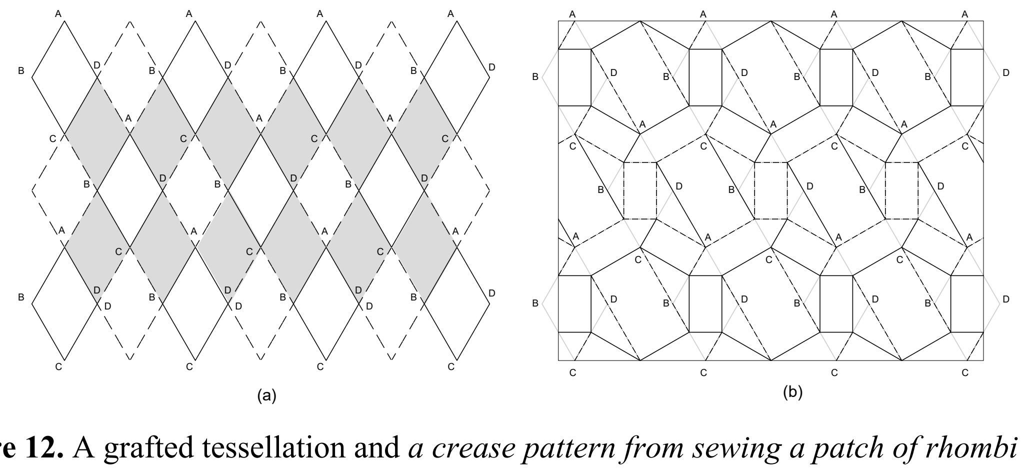


Figure 12. A grafted tessellation and *a crease pattern from sewing a patch of rhombi tiles.*

The above examples illustrate that for grafted tessellations to produce sewing patterns that can be sewn and pleated flat, the inserted polygons between the edges of original adjacent polygons must be either rectangles or parallelograms that are made of two congruent isosceles triangles. So, what kind of tessellation can be grafted using the technique shown in Figure 4? The technique used in Figure 4 can’t be applied to many other tessellations. Given the grafted tessellation in Figure 5(a) (redrawn as Figure 13(a)), a new tessellation (Figure 13(b)) that is the Archimedean tiling (3.6.3.6) of equilateral triangles and regular hexagons can be generated by eliminating the rhombi and bringing together the edges of the triangles and hexagons, or by eliminating the space in the sewing diagram in Figure 5(b). In Figure 13(c) the original rhombille tessellation has been overlaid with the (3.6.3.6) Archimedean tiling. It can be clearly observed that there is a direct relationship between them: each vertex of degree *n* in the rhombille tiling is at the center of an *n*-gon in the Archimedean tiling, and the edges in the (3.6.3.6) tiling emanating from a vertex are perpendicular to the edges of the polygon in the Archimedean tessellation that surrounds that vertex. The two tessellations are *duals*. One might then ask: Given any tessellation, is there always a "dual" tessellation in which corresponding edges have a consistent relationship, such as perpendicularity?

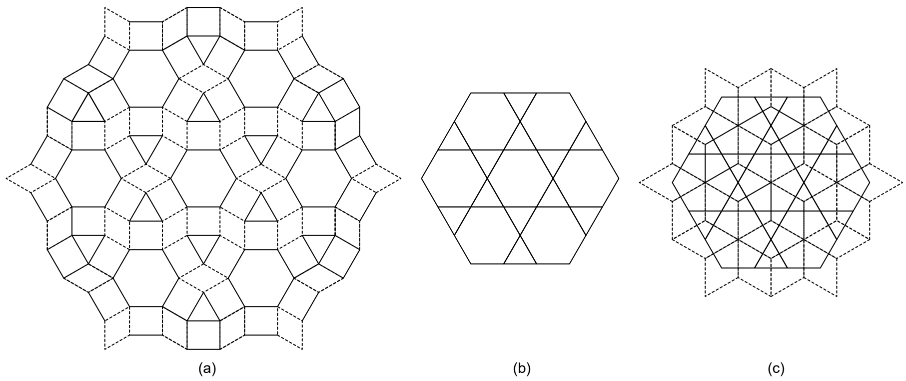


Figure 13. *A grafted tessellation, a new tessellation by eliminating the space in its sewing pattern, and a composition of this new tessellation and its original tessellation.*

6. Primary and Dual Relationship in Tessellation Grafting.

A given tessellation that we will call *primary* has vertices, edges, and tiles, and these can be understood as vertices, edges, and faces of a general plane graph. We denote its number of vertices by *V*, its number of edges by *E*, and its number of faces by *F*. An edge that belongs to only one face is a border edge, and we denote the number of border edges as *Eb*. An interior vertex is one that does not meet a border edge; we denote the number of interior vertices as *Vi*. The dual tessellation (or graph) can be understood as an interior dual graph that consists of *V′* vertices, *E′* edges, and *F′* faces and has a vertex for every face of the primary graph ($V' = F$), an edge for every nonborder edge of the primary graph ($E' = E - Eb$), and a face for every interior vertex of the primary graph ($F' = Vi$). In addition, the edges of the dual are perpendicular or parallel to the edges of the primary graph.

According to Lang (Lang 2018), James Clerk Maxwell (1831-79) found that the possibility of a primary graph having dual graph (which he called a reciprocal figure) depends on this difference: $2F - (E - Eb + 3)$. If the difference is 0, there is exactly one solution for the dual graph. If it is greater than zero, there is more than one solution, and if less than zero, no solution is possible unless for very special conditions.

To generate a grafted tessellation and the corresponding sewing pattern for fabric origami from a given tessellation, we can use Maxwell's condition to determine if a dual tessellation is possible. For example, a Star Octagram, shown in dashed lines in Figure 14(a) and demonstrated in (Rutzky & Palmer, 2011) has $F = 28$, $E - E_b = 48$, and therefore, $2F - (E - E_b + 3) = 5$. By Maxwell's condition, there are an infinite number of solutions for the dual graphs of the Star Octagram.

The algorithm for generating a sewing pattern from a Star Octagram (Figure 14(a)) is the following: Choose an arbitrary quadrilateral of the Star Octagram graph such as polygon HZAI and place a point A′ at an arbitrary location within it. Draw lines A′ B′ and A′ P′ of arbitrary lengths so that they are perpendicular to lines ZA, and IH. Draw lines A′I′ and B′I′ so that they are perpendicular to lines IA and AJ and meet at point I′. Repeat these steps until the dual graph is completed with 28 vertices and 48 edges, and each edge of the dual graph is perpendicular to an edge of the Star Octagram (Figure 14(b)). Next, a sewing pattern will be generated using the tessellation grafting technique. Cut along all edges of the Star Octagram and separate quads IHZA, AZBJ, and AJVI. Insert rectangles with the same width as the edge of triangle A′B′I′. Triangle A′B′I′ is formed by the edges of the inserted rectangles (Figure 14(c)). Repeat the same process with all the 28 quadrilateral faces of the Star Octagram. Delete all the original quadrilaterals and the rectangles to obtain the sewing diagram (Figure 14(d)). This sewing diagram is different from Chris Palmer's sewing diagram in (Rutzky & Palmer, 2011) since it is asymmetric.

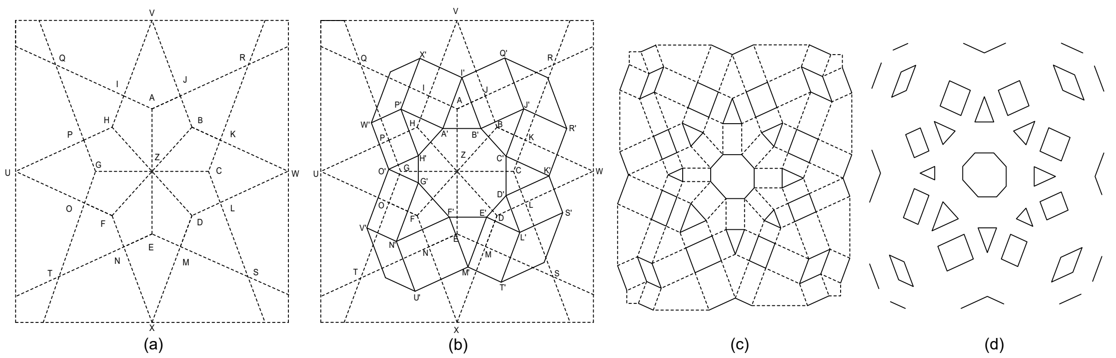


Figure 14. *A process of generating an asymmetrical sewing pattern for fabric origami based on Chris Palmer's Star Octagram. Figures 14(a) and (b) are enlarged for legibility.*

Since there are an infinite number of dual graphs of the Star Octagram, a symmetric design might be chosen to aesthetically reflect its Islamic-inspired 8-fold symmetric nature. Figure 15 shows two different symmetrical alternatives of fabric origami sewing patterns by grafting the tessellation differently. In each of the alternatives, the original Star Octagram tessellation and its dual graph, the grafted tessellation, the sewing pattern, and the sewn fabric origami are shown from left to right. Notice the slight differences in the final fabric origami pattern.

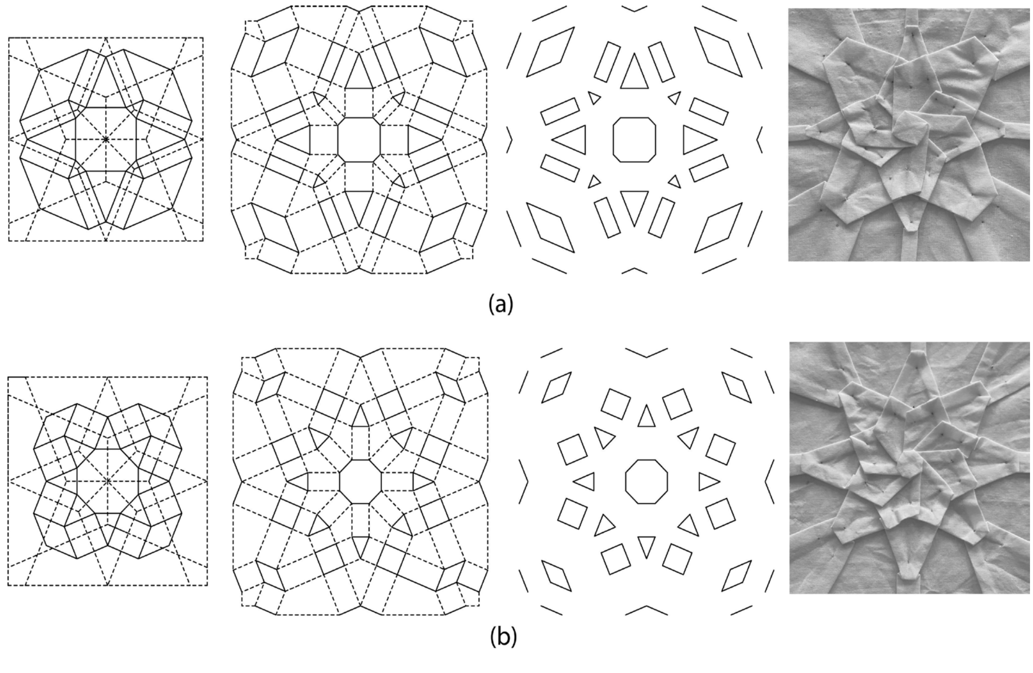


Figure 15. *Two alternatives of Star Octagram fabric origami. Two figures on the left are enlarged for legibility.*

7. Grafting Voronoi Tessellations

A Voronoi diagram (tessellation) is created by a scattered set ***S*** of points in the plane where each point *P* in ***S*** is surrounded by a polygonal region consisting of all points in the plane closer to *P* than to any of the other points in ***S***. If a point *P* in ***S*** is joined to a neighboring point *Q*, then all points equidistant from *P* and *Q* lie on the perpendicular bisector of the segment *PQ*. So the edges of the polygons in the Voronoi diagram are made up of segments of perpendicular bisectors of segments joining neighboring points. The graph of the segments joining neighboring points is called the Delaunay triangulation.

Figure 16 shows a Voronoi diagram in dashed outline with the Delaunay triangulation in solid outline superimposed. Note the Maxwell Condition is not satisfied as $2F-(E-Eb+3)=-1$. However, the edges of the Delaunay triangulation correspond orthogonally one to one with the edges of the Voronoi diagram, therefore, producing a desired dual tessellation in a particular condition. As a result, it can be used in tessellation grafting for fabric origami.

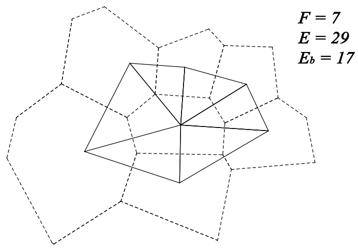


Figure 16. *A Voronoi diagram and its dual.*

Figure 17 shows an example of fabric origami based on a Voronoi tessellation. Figure 17(a) shows the Voronoi tessellation (in dashed lines), and Figure 17(b) shows the Voronoi with its dual Delaunay triangulation (in solid lines) embedded. Figure 17(b) shows the grafted tessellation and Figure 17(c) shows the sewing pattern. Figures 17(e) and 17(f) show the front and the back sides of the sewn fabric origami based on the sewing pattern 17(d). On the back side of the sewn fabric the seams of the fabric reflect the original Voronoi pattern. The front side of the fabric is carefully pleated and flattened to create a new tessellation.

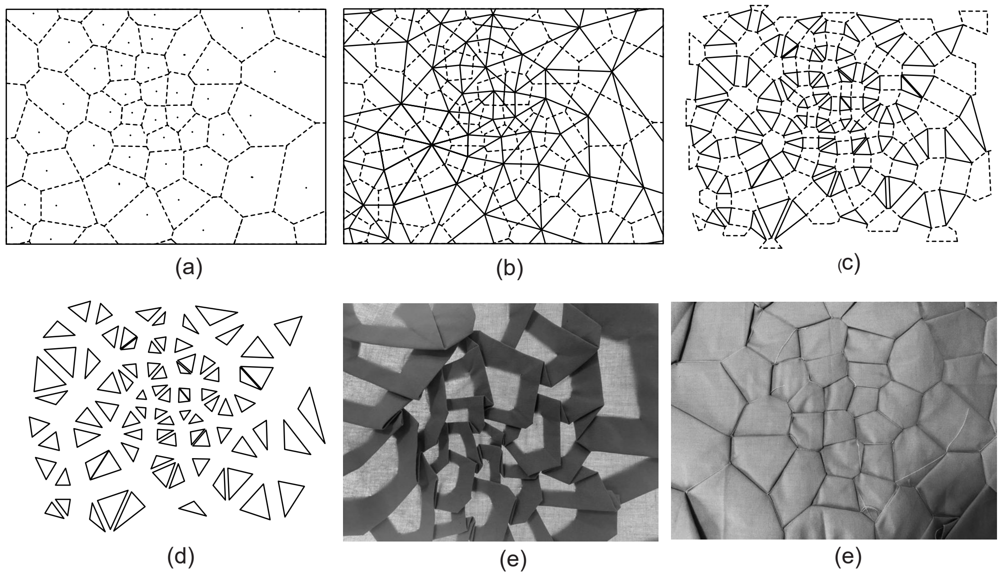


Figure 17. *(a) A Voronoi tessellation, (b) with its Delaunay triangulation, (c) its grafted tessellation, (d) its sewing pattern and (e) and (f) fabric origami. Figure 17(a) is enlarged for legibility.*

Since the perpendicular bisectors of any triangle meet at one point, one can find the *reciprocal figure* of any triangular mesh, a Voronoi tessellation all of whose vertices are 3-valent and in which corresponding lines are perpendicular bisectors of the edges of the mesh. Figure 18 shows an example of a triangular tessellation and its reciprocal figure, a heptakis heptagonal tiling, in which there are two hexagons and one heptagon at each vertex. This is based on a mapping of a (7.6.6) tiling in the hyperbolic plane into the Euclidean plane using a Poincaré mapping. The triangle tessellations are in solid line while the heptagonal/hexagonal tiles are in dashed line. Figure 18(a) shows the two reciprocal graphs together. Figure 18(b) shows the grafted tessellation, and Figure 18(c) shows the sewing pattern. Both Figures 18(b) and (c) are altered from the original tessellation in Figure 18(a) by combining the seven triangles in the center into a heptagon for aesthetic reasons. Figures 18(d), (e), and (f) show the front, the back, and the detail of the sewn fabric origami based on the sewing pattern in Figure 18(c).

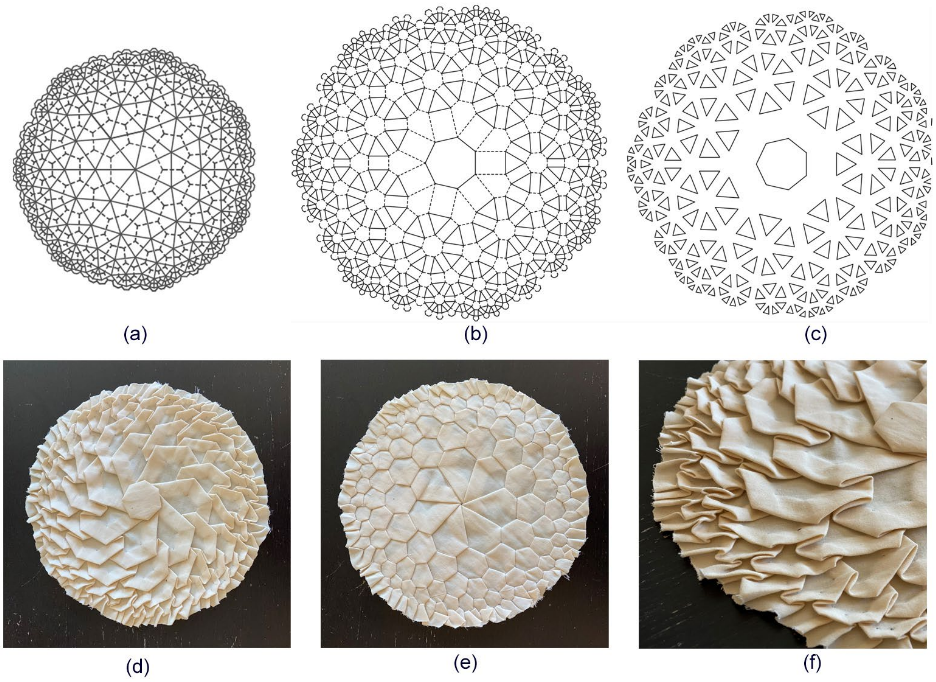


Figure 18. *A triangular tessellation together with its Voronoi diagram, its grafted tessellation, sewing pattern and fabric origami. Figure 18(a) is enlarged for legibility.*

There is a simple algorithm to graft a Voronoi tessellation, since each of its cells (polygon tiles) is defined by a single point. When each of Voronoi cells is scaled consistently from its defining point, the spaces created between the cells are rectangles and triangles, producing the same result as grafting a Voronoi tessellation. The relationship between the size of the original fabric and the finished fabric origami when working with Voronoi tessellations is also very clear. Given a Voronoi diagram as shown in Figure 19(a), if each of the Voronoi cells is scaled from its defining point using a scale factor of 0.5 to create the grafted tessellation, the linear dimensions of the finished fabric origami will be 0.5 of the original fabric (Figure 19(b)). Figures 19(c) and(d) show that the linear dimensions of the finished fabric origami will be 65 percent and 85 percent of the original fabric if the Voronoi cells are scaled using scale factors of 0.65 and 0.85 respectively.

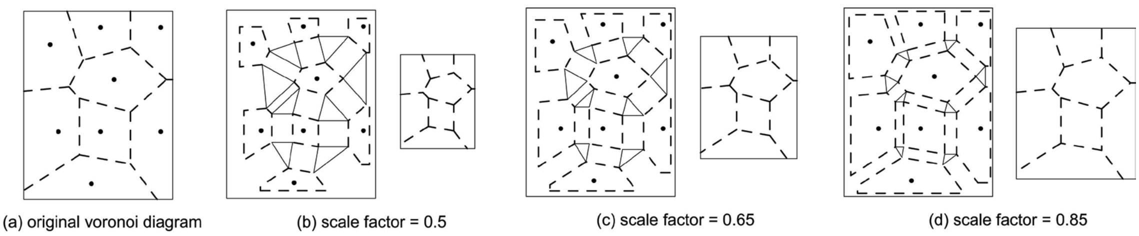


Figure 19. *The relationship between the original fabric size and the finished fabric origami.*

8. Two Case Studies for Fabric Origami Artworks

What is the mathematical and algorithmic strategy an artist can use to come up with sewing patterns using the tessellation grafting technique for fabric origami? How do artists start to create fabric origami based on conceptual inspirations? I will present two case studies to demonstrate how I create fabric origami artworks. The first piece is entitled "Dawn," a fabric origami art with watercolor added to bring attention to the folded design. The second piece is entitled "Kallos," a wearable fabric origami artwork.

Figure 20. *Dawn, 33"x19", Muslin Cotton, 2022. Artist Jiangmei Wu.*

The concept for "Dawn" was inspired by a logarithmic spiral that is expressed through the density and scale of the folded patterns of a fabric origami tessellation and the subtle changes of the watercolor hues (Figure 20). To create this artwork, I started with a simple logarithmic spiral (Figure 21(a)) then overlaid a triangle mesh, adjusting and refining it so that the triangles get smaller the closer they are to the origin of the spiral (Figure 21(b). The technique is based on a simple algorithm used for mesh refinement and coarsening to adaptively represent terrains (Suárez & Plaza, 2008). To add aesthetic interest, I distorted the mesh in certain areas to provide some visual contrast. A Voronoi tessellation was then generated from the triangle mesh (Figure 21(c)). This Voronoi tessellation was then grafted by inserting each of the corresponding triangles at each of the vertices (Figure 21(d)). A sewing pattern for fabric origami was obtained by removing all the inserted polygons in the grafted tessellation. After the sewing pattern was transferred to the fabric, the points of each triangle were stitched together on one side of the fabric. On the other side of the fabric, the gatherings of the fabric were then pressed flat to create the folding patterns. Once the fabric was pleated, watercolor was then applied to the fabric.

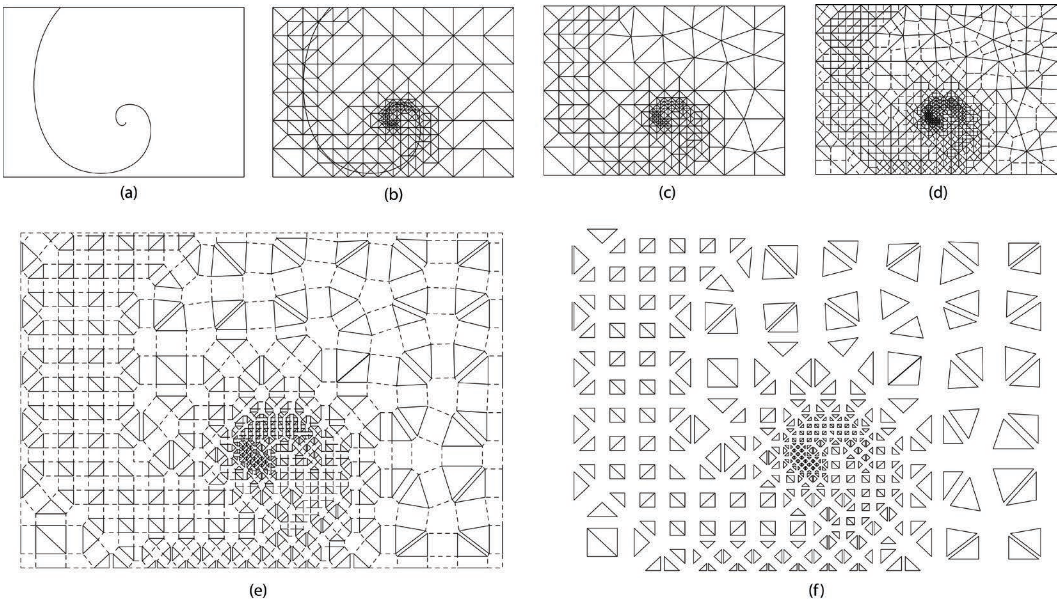
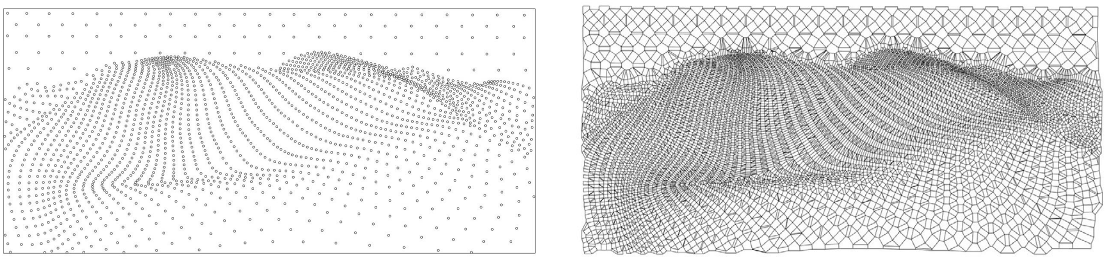


Figure 21. *Process of creating "Dawn."*

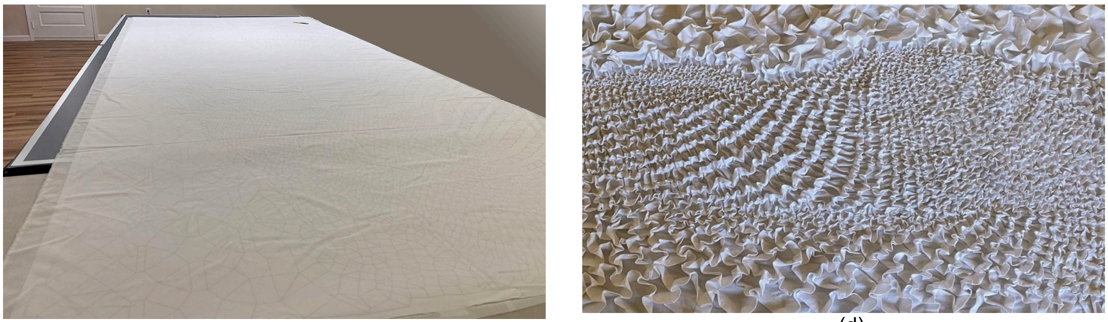
"*Kallos"* is a wearable fabric origami artwork. *Kallos* is a Greek word for beauty (Figure 22). The work pays homage to natural folds and folds of the worlds—how various cultures use folds, pleats, wraps, and drapes in clothing (for example, in contemporary fashion Alexander McQeen's Oyster dress). *Kallos* explores the connections between folds, body, material, and movement. As a wearable wrap made from multiple small folds, *Kallos* embraces the volume and depth of the folds, and the natural silhouettes of the folded material as it drapes over the body. It provides a sensual feel and appearance for the wearer. The wearer's body glides through the world, separated only by this second skin of folds and pleats.

Figure 22. *Kallos, width 60", height 30", Muslin Cotton, 2023. Artist Jiangmei Wu. Modelled by Hiroko Hanahuma.*

To create the undulating landscape-like design, I started by drawing a point cloud in a computer program (Figure 23(a)). This undulating design took into consideration contrasting patterns of the wrap and how the patterns look as the fabric wraps and drapes around the body. The point cloud was then used to generate the Voronoi tessellation, which was subsequently used to generate tessellation grafting patterns for sewing (Figure 23(b)). This sewing pattern was then transferred to the back side of the fabric that was about 120 inches by 60 inches (Figure 23(c)). After sewing over two thousand triangles on the back side of the fabric, the front side of the fabric was gathered systematically (Figure 23(d)), thus reducing the fabric to about 60 inches by 30 inches. The fabric was then carefully pleated and hot ironed. To make pleating and folding more manageable, I wet the fabric slightly, using a technique similar to wet folding.



(a) (b)



(c) (d)

Figure 23. *Design process for Kallos.*

9. Conclusion

In this article I have discussed the processes and techniques of how I create fabric origami. I have also demonstrated a few examples of algorithms that are used to generate sewing patterns for fabric origami and flat-foldable crease patterns from sewing patterns. The tessellation grafting technique focuses on a specific relationship, namely, that corresponding edges of the original tessellation and its dual are perpendicular. Can we consider other types of reciprocal relationship such as parallelism? In addition to the Voronoi diagram and its dual, what other particular types of duals include tessellations with vertices that are not 3-valent, but 4, 5, or 6-valent?

The process of making fabric origami can be highly technical and time-consuming. By adding a dimension of mathematical complexity and technical skills into what is traditionally viewed as a "women's craft," I hope to continue to create, and encourage others to create, more fabric origami. My goal is to help blur the boundaries between high art and low art, between the domestic and the public spheres, and between traditionally perceived male and female roles.

Acknowledgments

The author's interest and work on fabric origami is seeded from a collaborative grant supported by Center for Craft that is based in Ashville, North Carolina. Further support is provided by the Office of the Vice President for Research and the College of Arts and Humanities Councils, Indiana University, Bloomington. Some portions of this paper have been adapted from two articles in the *Proceedings of Bridges: Mathematics, Art, Music, Architecture, and Culture (Wu, 2022, 2024).* The author extends heartfelt thanks to Doris Schattschneider for her meticulous edits and invaluable insights, as well as to Chunfeng Huang and Matthias Webber for their thoughtful discussions.

References:

Bateman, A. (2007). *Tess: Origami tessellation software*. Retrieved November 27 from http://www.papermosaics.co.uk/software.html

Burns, A. (2022). Rethinking Fabric: The Application of Fabric Manipulation Techniques in Fashion Design Education. *International Journal of Art & Design Education*, *41*(1), 66-80. https://doi.org/https://doi.org/10.1111/jade.12375

Fujimoto, S., & Nishiwaki, M. (1982). Seizo Soru Origami Asobi no Shotai (Creative Invitation to Paper Play). In T. A. C. Center (Ed.).

Gjerde, E. (2009). *Origami tessellations : awe-inspiring geometric designs*. A K Peters. Table of contents only http://www.loc.gov/catdir/toc/ecip0718/2007020769.html

Publisher description http://www.loc.gov/catdir/enhancements/fy0730/2007020769-d.html

Justin, J. (1986). Mathematics of origami, part 9. *British Origami*, *June 1986*.

Justin, J. (1994). Mathematical remarks about origami bases. *Symmetry: Culture and Science*, *5*(2), 153-165.

Kawasaki, T. (1989). On the relation between mountain-creases and valley-creases of a flat origami. (Ed.),^(Eds.). First International Meeting of Origami Science and Technology, Dipartimento di Fisica dell'Università di Padova, Padova, Italy.

Kawasaki, T., & Yoshida, M. (1988). Crystallographic Flat Origamis. *Memoirs of the Faculty of Science, Kyushu University, Series A, Mathematics XLII*(2), 153-157.

Lang, R. J. (2018). *Twists, tilings, and tessellations : mathematical methods for geometric origami*. CRC Press, Taylor & Francis Group.

Momotani, Y. (1984). Wall. (Ed.),^(Eds.). BOS Convention 1984 Autumn, London.

Museum, C. H. S. D. (2013). *Reiko Sudo on Origami Textiles*. Retrieved July 2nd from https://www.cooperhewitt.org/tag/reiko-sudo/

Palmer, C. K. (1997). *Extruding and Tessellating Polygons from a Plan*. Second International Meeting of Origami Science and Scientific Origami, Shiga, Japan: Seian Univerisity of Art and Design.

Rutzky, J., & Palmer, C. K. (2011). *Shadowfolds : surprisingly easy-to-make geometric designs in fabric*. Kodansha International.

Suárez, J. P., & Plaza, A. (2008). Four-triangles adaptive algorithms for RTIN terrain meshes. *Mathematical and Computer Modelling*, *49*(2009), 1012-1020.

Verrill, H. (1998). *Origami Tessellations*. Bridges: Mathematical Connections in Art, Music, and Science, Southwestern College, Winfield, Kansas. http://archive.bridgesmathart.org/1998/bridges1998-55.html

Wolff, C. (1996). *The art of manipulating fabric*. Chilton Book Co. Publisher description http://www.loc.gov/catdir/enhancements/fy1502/96032201-d.html

Wu, J. (2022). *Grafting Tessellations for Fabric Origami*. Bridges Aalto 2022, Aalto University, Espoo, Finland.

Wu, J. (2024). *Algorithms for Generating Crease Patterns from Sewing Patterns*. Bridges Richmond 2024, Virginia Commonwealth University.